Some Definitions

- \( N^S \) = Detected pairs with single layer target.
- \( N^M \) = Detected pairs with multi-layer target.
- \( N^B \) = Background pairs.
- \( N^C \) = Coulomb pairs.
- \( N^{NC} \) = Non-Coulomb pairs.
- \( n^A_S \) = Broken atoms in single-layer.
- \( n^A_M \) = Broken atoms in multi-layer.
- \( N^A \) = Created atomic pairs.
Some Relations

- The number of background pairs (Coulomb and Non-Coulomb) is the same in the two targets.
- Of course: \( N^B = N^C + N^{NC} \)
- The number of atomic pairs is also the same in the two targets.
- The number of broken pairs differs and are given by:
  
  \[ n_s^A = P^S N^A \]
  
  \[ n_m^A = P^M N^A \]

where \( P^S \) and \( P^M \) are the breakup probabilities of pionium in the single and multi layer targets respectively.
The Main Relation

\[ N^B = \frac{P^S N^M - P^M N^S}{P^S - P^M} \]

can be easily proven if we consider:

\[ N^S = N^B + n^A_S = N^B + P^S N^A \]

\[ N^M = N^B + n^A_M = N^B + P^M N^A \]

The relation can be equivalently expressed as:

\[ N^B = N^S - \frac{P^S}{P^S - P^M} (N^S - N^M) \]

or

\[ N^B = N^M - \frac{P^M}{P^S - P^M} (N^S - N^M) \]
The Main Idea

$P^S$ and $P^M$ depend on the lifetime and we ignore their value. However, we can use some test values $P_0^S$ and $P_0^M$ and compute the errors. As an example we have used:

$$P_0^S = P^S(\tau = 3fs) = 0.454$$

$$P_0^M = P^M(\tau = 3fs) = 0.231$$
The Systematic Error

We want to study wether:

\[ N_0^B = \frac{P_0^S N^M - P_0^M N^S}{P_0^S - P_0^M} \]

is a good estimate of \( N^B \).

The systematic error would be:

\[ N^B - N_0^B = (N^S - N^M) \times \frac{P_0^S P^M - P_0^M P^S}{(P^S - P^M)(P_0^S - P_0^M)} \]

if we assume \( N^{NC} \approx 0 \) \(^a\) and consider \( N^A = kN^C \) we have \( (N^C = N^B) \):

\[ \frac{N^B - N_0^B}{N^B} = k \frac{P_0^M P_0^S - P^M P^S}{P_0^S - P_0^M} \]

\(^a\)Non Coulomb pairs are 2% of the background in the \( Q < 2MeV/c \) region.
The Statistical Error

The Statistical Error in the calculation of background with the main Formula is given by:

\[ \sigma_{NB} = \frac{\sqrt{(P^S)^2 N^M + (P^M)^2 N^S}}{P^S - P^M} \]

Notice that \( P^M < P^S \), in particular, around \( \tau = 3f_s P^M \approx P^S/2 \). This means that the statistics in the multi-target layer contributes larger to the statistical error. In particular, if we assume \( N^{NC} \approx 0 \) then:

\[ \frac{\sigma_{NB}}{NB} = \frac{1}{\sqrt{NB}} \times \frac{\sqrt{(P^S)^2 + (P^M)^2 + kP^SP^M(P^S + P^M)}}{P^S - P^M} \]
Two Cases

We have analyzed two particular cases in the $F < 2$ region $^a$:

- $N_C = 15000$, accumulated statistic of the single layer target 2001.
- $N_C = 5500$, accumulated statistic of the multi-target layer 2002.

We have used $k = 0.69$ for the $k$ factor.

<table>
<thead>
<tr>
<th>Errors</th>
<th>Stat. $(3, f/s)$</th>
<th>Sys. $(2.4, f/s)$</th>
<th>Sys. $(3.6, f/s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_C = 15000$</td>
<td>2.0%</td>
<td>1.9%</td>
<td>$-1.7%$</td>
</tr>
<tr>
<td>$N_C = 5500$</td>
<td>3.4%</td>
<td>1.9%</td>
<td>$-1.7%$</td>
</tr>
</tbody>
</table>

$^a$The region with atomic pairs contamination.
Errors as a Function of $\tau$

- Systematic error $N_C^S (F<2)$
- Statistical error $N_C^S (F<2)=15000$
- Statistical error $N_C^S (F<2)=5500$
Second approach

We have started a second approach to the Main Formula by analyzing the magnitude:

\[ \delta P^1 = \frac{N^S - N^M}{kN_0^C} \]

\[ = \frac{(P_0^S - P_0^M)(N^S - N^M)}{k(P_0^S N^M - P_0^M N^S)} \]
**Second approach (2)**

We have not computed the possible transmission of errors, so, the result should be considered as preliminarily:

<table>
<thead>
<tr>
<th>Errors</th>
<th>Sys. $\tau$</th>
<th>Sys. $(2.4 fs)$</th>
<th>Sys. $(3.6 fs)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2nd app.</td>
<td>$-0.36%$</td>
<td>$0.34%$</td>
<td></td>
</tr>
</tbody>
</table>

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### Diagram

Statistical error $N^C(Q<2\text{ MeV/c})$  
Systematic error $N^C(Q<2\text{ MeV/c})$ 2nd app.

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**Notes:**

- We have not computed the possible transmission of errors, so, the result should be considered as preliminarily.
- The table shows the systematic error for different approaches.
- The diagram illustrates the statistical and systematic errors for $N^C(Q<2\text{ MeV/c})$ and the 2nd approach.